Quantum Electronics **27** (8) 727 – 731 (1997)

PACS numbers: 42.65.Hw; 42.40.Eq

Spatial structure of holographic gratings in photorefractive crystals with a nonlocal response

S A Bugaĭchuk, A G Kutana, A I Khizhnyak

Abstract. The structure and spatial positions of dynamic gratings responsible of steady-state four-wave mixing are determined for nonlinear media with a nonlocal response. It is shown that an analytic solution found for a double phase-conjugate mirror is fundamental and that it can be used to obtain solutions for such a mirror when two pump beams are present.

1. Introduction

Four-wave mixing (FWM) in dynamic transmission gratings (Fig. 1) formed in photorefractive crystals (PRCs) with a nonlocal response has been attracting interest for a fairly long time [1 – 4]. A theoretical analysis of steady-state FWM can be based on the solution of a system of coupled-wave equations, from which it follows — as demonstrated by formula (3.56) in Ref. [2] — that the amplitude of a dynamic grating changes with the depth in a PRC and that the grating lines are directed along the bisector of the angle of convergence of the beams which generate the grating (Fig. 1).

A steady state established after the arrival of beams *1*, *2*, and *3* in a PRC (Fig. 1), which does not initially contain a refractive-index grating, has been investigated in detail [1 – 4]. The difference between the phases of the interacting waves ($\Phi - \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4$) can assume two possible values: 0 or π. These two values correspond to the formation of dynamic gratings identical in structure but shifted by half their period relative to one another. The solutions for these two values of Φ differ only in the sign of the amplitude of the fourth wave which is generated. Therefore, in an analysis of FWM in a PRC it is usual to consider the specific case when $\Phi = 0$.

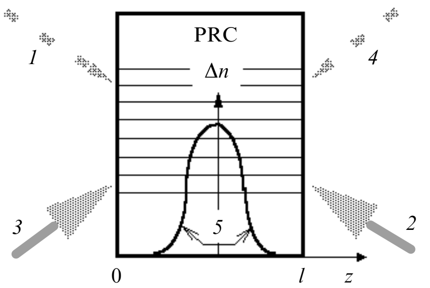

Figure 1. Schematic diagram of four-wave mixing in a photorefractive crystal with a transmission grating: (*1 – 4*) interacting beams; (*5*) curve demonstrating localisation of a dynamic grating (Δn) in the presence of a two-sided phase-conjugate mirror and of beams *3* and *2* of equal intensities when $\gamma l \gg 1$.

S A Bugaĭchuk, A G Kutana, A I Khizhnyak International Centre 'Institute of Applied Physics', National Academy of Sciences of Ukraine, Kudryavskaya ul. 10-G, 254053 Kiev, Ukraine

Received 19 February 1997
Kvantovaya Elektronika **24** (8) 746 – 750 (1997)
Translated by A Tybulewicz

Cronin-Golomb et al. [1] obtained two more solutions for which the phase difference Φ assumes the values $\Phi(0) = \pi$ and $\Phi(l) = 0$ at the opposite boundaries of a PRC for these solutions, the energy exchange efficiency should be less than the efficiency of the traditional energy exchange. It was found later [5] that one of these solutions is unstable.

The main effort has been concentrated so far on an analysis of the characteristics of the interacting waves, but hardly any attention has been given to the structure of a dynamic grating responsible for FWM, although it is the grating that determines the whole energy exchange process. For this reason, our aim will be to investigate the structure of dynamic gratings formed in a PRC under FWM conditions.

We shall obtain an analytic solution describing the structure of dynamic gratings, which makes it possible to analyse steady states, to propose ways of attaining such states, and to show that the solution for a double phase-conjugate mirror (DPCM) is fundamental: this solution can be used to derive general solutions, i.e. when all beams are present at the entry to a PRC [we recall that in a DPCM the process of phase conjugation of beams *3* and *2* occurs in the absence of beams *1* and *4* (Fig. 1) at the PRC faces through which they enter].

2. Equations describing the structure of a dynamic grating

The system of coupled-wave equations for steady-state FWM in a PRC with a nonlocal diffusion-type response can be written as follows [1 – 4]:

$$\begin{aligned}
\frac{dA_1}{dz} &= \gamma(A_1A_3 + A_2A_4\cos\Phi)A_3\ , \\
\frac{dA_3}{dz} &= -\gamma(A_1A_3 + A_2A_4\cos\Phi)A_1\ , \\
\frac{dA_2}{dz} &= \gamma(A_1A_3\cos\Phi + A_2A_4)A_4\ , \\
\frac{dA_4}{dz} &= -\gamma(A_1A_3\cos\Phi + A_2A_4)A_2\ ,
\end{aligned} \tag{1}$$

where $A_n \exp(i\varphi_n) = E_n\sqrt{J}$ is the normalised complex amplitude of the nth wave; γl is the coupling constant of the PRC; l is the PRC length (Fig. 1); $J = J_1 + J_2 + J_3 + J_4 = \mathrm{const}$ is the total intensity of the interacting beams.

As pointed out above, the steady-state phase difference $\Phi = \varphi_1 + \varphi_2 - \varphi_3 - \varphi_4$ can assume one of two values: 0 or π.

We can easily see that, if $\Phi = \pi$, the replacement of A_4 with $-A_4$ transforms the system of equations (1) into the system for $\Phi = 0$. Therefore, it is usual to solve the system (1) on the assumption that $\Phi = 0$ [1–4]. It was shown in Ref. [2] that one can go over from the system of equations (1) to the following equation describing the behaviour of a dynamic grating:

$$\frac{\mathrm{d}^2 \ln B}{\mathrm{d}z^2} = -4\gamma^2 B^2 , \tag{2}$$

where

$$B = (A_1 A_3 + A_2 A_4 \cos\Phi) . \tag{3}$$

The solution of Eqn (2) is

$$B = \frac{C}{\cosh(2\gamma z C + b)} \tag{4}$$

and it contains two constants of integration: b and

$$C = \tfrac{1}{2}(U^2 + 4B^2)^{1/2} = \mathrm{const} ,$$

where

$$U = A_3^2 + A_4^2 - A_1^2 - A_2^2 .$$

The envelope of a grating described by expression (4), represented by curve *5* in Fig. 1, is a stationary soliton. Its maximum is located in the section $z = -b/2\gamma C$ and, the higher the value of C, the narrower is the distribution of this soliton. Since B is a real quantity, we shall introduce a new variable

$$u(z) = \gamma \int_0^z B(z')\,\mathrm{d}z' . \tag{5}$$

The system of equations (1) then splits into two systems, of two equations each, for waves *1*, *3*, and *4*, *2*, respectively. The solutions of these two systems of equations are found readily:

$$\begin{aligned} A_{1,3} &= D_{1,2}\cos u \pm D_{2,1}\sin u , \\ A_{2,4} &= E_{1,2}\cos u \pm E_{2,1}\sin u . \end{aligned} \tag{6}$$

Here, the plus sign corresponds to the first subscript and the minus sign to the second subscript; the amplitudes E_n and D_n are found from the boundary conditions. The solution described by the set of expressions (6) readily yields the characteristics of energy exchange between the interacting beams, for example, the dependence of the phase-conjugate reflection coefficient [$R_{\mathrm{m}} = J_4(0)/J_3(0)$] on the coupling constant γl, plotted for various values of $J_1(0)$ in Fig. 2.

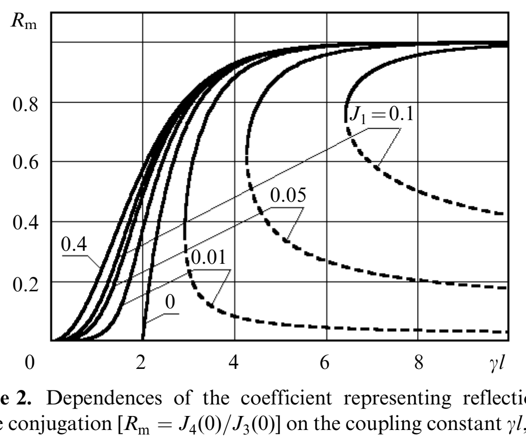

Figure 2. Dependences of the coefficient representing reflection with phase conjugation [$R_{\mathrm{m}} = J_4(0)/J_3(0)$] on the coupling constant γl, plotted for different values of $J_1(0)$.

If $J_1(0) = 0$, when a DPCM is formed, the threshold of its appearance can be seen clearly ($\gamma l = 2$). The curve describing a DPCM is a boundary separating two classes of solutions. The curves for $J_1(0) \neq 0$, located to the left of this boundary, represent solutions of class 1 for which the phase difference is $\Phi = 0$ over the whole thickness of the PRC. The situation is more complex for solutions of class 2 [$\Phi(0) = \pi$]. The latter exist if $\gamma l > 2$ and $J_1(0) \neq 0$, and they are represented by loop curves located to the right and below the boundary curve of the DPCM. Solutions of class 2 are double-valued [1]. Our analysis and other investigations [5] show that the lower branches of the loops describe unstable solutions and the upper branches correspond to stable solutions.

We shall consider in more detail these two classes of solutions and find the structure of the dynamic gratings that correspond to these two classes. We shall begin with the situation at the boundary corresponding to a DPCM.

3. Characteristics of a double phase-conjugate mirror

A DPCM appears in a PRC when only two beams, *3* and *2* (Fig. 1), are incident on this crystal. Solution of Eqn (5) can then be represented in the form

$$\begin{aligned} &J_1(l) = J_3(0)\eta , \quad J_3(l) = J_3(0)(1-\eta) , \\ &J_2(0) = J_2(l)(1-\eta) , \quad J_4(0) = J_2(l)\eta , \end{aligned} \tag{7}$$

where $\eta = \sin^2 u(l)$ is the diffraction efficiency of a dynamic grating. Introducing $x = \exp(2C\gamma l)$ and $y = \exp b$, we obtain the following equations for finding x, y, and η:

$$\frac{x-1}{x+1} = \frac{\ln x}{\gamma l} , \tag{8}$$

$$y^2 = \frac{1 - J_{23}x}{x(J_{23} - x)} , \tag{9}$$

$$\eta = \frac{(1 - J_{23}x_m)(J_{23} - x_m)}{J_{23}(1 + x_m)^2} , \tag{10}$$

where $J_{23} = J_2(l)/J_3(0)$ is the ratio of the intensities of the beams incident on the PRC and x_m are the roots of Eqn (8), governed only by the coupling constant γl.

If $\gamma l \leqslant 2$, Eqn (8) has only one solution ($x_1 = 1$) characterised by $\eta_1 \leqslant 0$, i.e. a DPCM is not formed. The second root, $x_2 > 1$, appears only for $\gamma l > 2$ and this root is characterised by $0 \leqslant \eta_2 \leqslant 1$. We then find that $\ln x_2 \to \gamma l c$ with increase in γl, and that $\eta_2 \to 1$ if $J_{23} = 1$. For $J_{23} = x_2$ or $1/x_2$, the parameter η_2 vanishes, i.e. the range of existence of the solution with a DPCM is limited to $1/x_2 < J_{23} < x_2$. It follows from Eqns (8) and (10) that η_2 is governed by two parameters γl and J_{23}. Replacement of J_{23} with $1/J_{23}$ does not alter η_2 (Fig. 3a). The maximum of η_2 is reached at $J_{23} = 1$.

The structure of a dynamic grating is described by the following expression:

$$B(z) = \ln x_2 \left\{ 2\gamma l \cosh\left[\ln x_2 \left(\frac{z}{l} - \frac{1}{2} + \frac{\ln[(1 - J_{23}x_2)/(J_{23} - x_2)]}{2\ln x_2} \right)\right]\right\}^{-1} . \tag{11}$$

We can see that, if $J_{23} = 1$, the grating is localised exactly in the middle of the PRC. If $J_{23} \neq 1$, the grating is shifted away from the centre in the direction of the beam with a lower

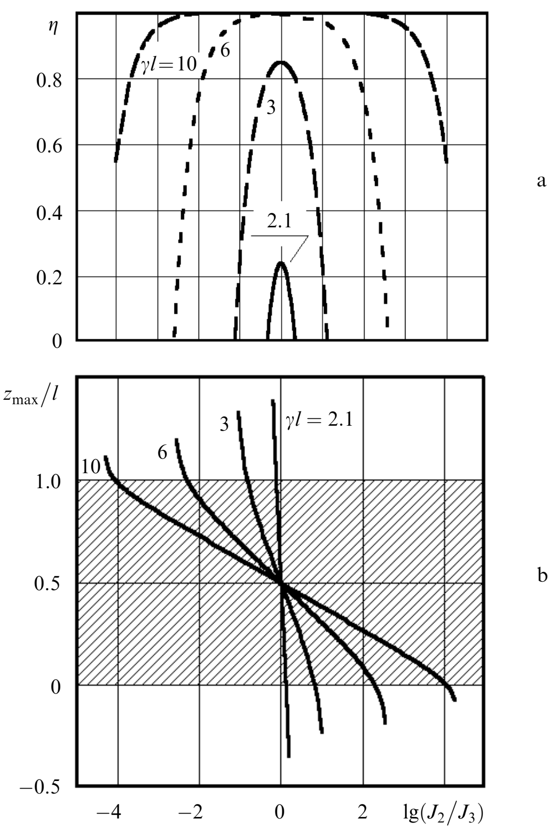

Figure 3. Dependences of the diffraction efficiency η of a dynamic grating in the presence of a double phase-conjugate mirror (a) and of the positions of the maximum of its distribution (b) on the ratio of the intensities of the grating-forming beams, plotted for different coupling constants γl. The region occupied by the photorefractive crystal is shown shaded.

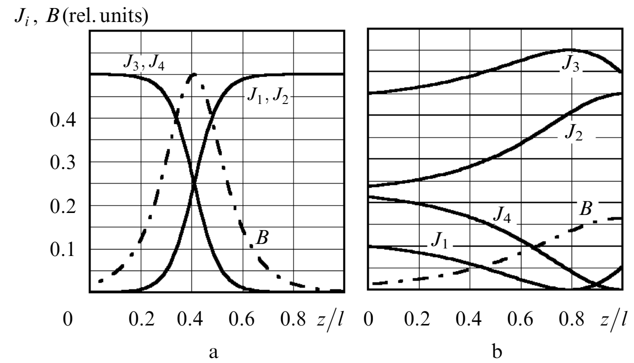

Figure 4. Variations, across the thickness of a photorefractive crystal, of the intensities of four interacting beams J_i and of the envelope of a dynamic grating B for class 1 (a) and class 2 (b) solutions.

intensity (Fig. 3b). However, if J_{23} lies outside the range

$$\frac{2x_2}{x_2^2+1} \leqslant J_{23} \leqslant \frac{x_2^2+1}{2x_2},$$

the maximum of the grating described by expression (11) is outside the crystal and as J_{23} approaches the boundaries of the range of existence of the DPCM, the grating reflection maximum is shifted to infinity and this suppresses the mirror. It follows from Fig. 3b that variation of the ratio of intensities of beams *3* and *2* can be used to control the position of a dynamic grating in a PRC.

4. Solution for $J_1(0) \neq 0$

Fig. 4 shows how the intensities of the interacting beams change with the depth of a PRC for two classes of the steady-state solutions mentioned in Section 3. The traditional energy transfer from beams *3* and *2*, to beams *1* and *4*, respectively, occurs for the class 1 solution (Fig. 4a). The class 2 solution demonstrates a more complex energy exchange. Beam *3* is amplified, depleting totally the energy of beam *1*, in the front part of the crystal ($0 \leqslant z \leqslant l_{\text{sh}}$). The reason for this nontraditional direction of energy transfer is the presence of beam *4* at the boundary $z = l_{\text{sh}}$ [$J_4(l_{\text{sh}}) \neq 0$], which makes it possible to satisfy the condition $\Phi(0) = \pi$. A DPCM forms in the rest of the crystal ($l_{\text{sh}} \leqslant z \leqslant l$) and the energy from beam *3* returns in part to beam *1*. In contrast to the nonmonotonic energy exchange between beams *1* and *3*, the transfer of energy from beam *2* to beam *4* is unidirectional, but it is less efficient than in the case of the class 1 solutions.

These features are expected because, for the class 2 solutions, Φ assumes two different values at the opposite faces of the PRC, whereas it follows from the initial equations that Φ should be constant. This contradiction is resolved by matching, in the $z = l_{\text{sh}}$ section, two solutions with $\Phi = \pi$ (in the part of the crystal where $0 \leqslant z \leqslant l_{\text{sh}}$) and $\Phi = 0$ ($l_{\text{sh}} \leqslant z \leqslant l$).

It follows from the system of equations (1) that evolution of the waves interacting in the crystal is subject to the condition of conservation of the total intensity of the concurrent beams, i.e. $J_1(z) + J_3(z) = J_{\Sigma 1} = \text{const}$ and $J_2(z) + J_4(z) = J_{\Sigma 2} = \text{const}$. Then, if both waves *3* and *1* (Fig. 1) are present at the entry to the crystal, then for the class 1 solutions we can imagine that our crystal grows to the left of the $z = 0$ face reaching a thickness l_{sh} such that $J_1(-l_{\text{sh}}) = 0$. A DPCM with $J_3(-l_{\text{sh}}) = J_{\Sigma 1}$ forms in such a lengthened crystal of thickness $l + l_{\text{sh}}$. For the class 2 solutions, a mirror forms in a shortened crystal of thickness $l - l_{\text{sh}}$ and we have $J_3(l_{\text{sh}}) = J_{\Sigma 1}$. Solutions of both classes can then be found simply by shifting the boundary l_{sh} and by using the solution for a DPCM.

The above analysis makes it possible to obtain readily the equations for finding the roots x_m and the shift l_{sh}:

$$\frac{x-1}{x+1} = \frac{\ln x \mp \ln Y}{\gamma l}, \tag{12}$$

$$l_{\text{sh}} = l\,\frac{\ln Y_m}{\ln x_m \mp \ln Y_m}, \tag{13}$$

where

$$Y = \left[1 + \left(\frac{J_1 x}{J_3}\,\frac{x - J_{23}}{J_{23}x - 1}\right)^{1/2}\right]\left[1 - \left(\frac{J_1}{J_3 x}\,\frac{J_{23}x - 1}{x - J_{23}}\right)^{1/2}\right]^{-1};$$

$J_{23} = J_{\Sigma 1}/J_2(l)$; Y_m is the value of Y at $x = x_m$. In Eqns (12) and (13) and subsequently the upper sign applies to $\Phi = 0$ and the lower to $\Phi(0) = \pi$. We can easily see from the definition of Y that, if $J_1 \to 0$, then also $l_{\text{sh}} \to 0$ when Eqn (12) reduces to Eqn (8).

Having found x_m, we can now determine η from Eqn (10) and the intensity of beam *4* from the following expression, which is deduced from the general solution given by definition (5):

$$J_4(0) = J_2(l)\left\{\left[\frac{(1-\eta)J_1(0)}{J_{\Sigma 1}}\right]^{1/2} \mp \left[\eta\left(1 - \frac{J_1(0)}{J_{\Sigma 1}}\right)\right]^{1/2}\right\}^2. \tag{14}$$

The structure of a dynamic grating B is still described by expression (11), but subject to the substitutions $z \to z \pm l_{\mathrm{sh}}$, $l \to l \pm l_{\mathrm{sh}}$, $x_2 \to x_m$, $J_{23} = J_{\Sigma 1}/J_2(l)$. Fig. 5 shows the dependences of the position of the grating envelope maximum on the intensity of beam *1* for the two classes of solutions when the total intensity of beams *1* and *3* is equal to the intensity of beam *2*, and the coupling constant is $\gamma l = 10$. It is clear from Fig. 5 that the grating position is more sensitive to the ratio of the interacting beam intensities than in the case of a DPCM (Fig. 3b).

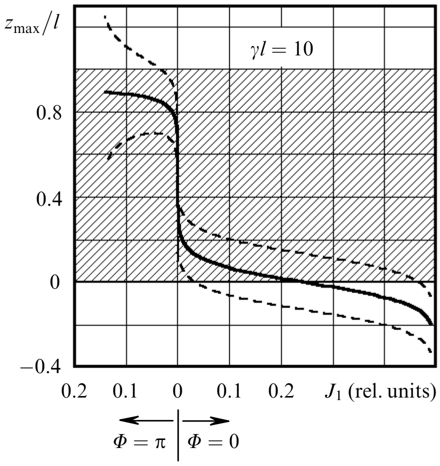

Figure 5. Dependences, on the intensity of beam *1*, of the position of the maximum of the dynamic grating envelope (continuous curve) and of the points at half the maximum (dashed curves) for the class 1 ($\Phi = 0$) and class 2 ($\Phi = \pi$) solutions. The region occupied by the photorefractive crystal is shown shaded.

If $\Phi(0) = 0$ and $J_1(0) \neq 0$, the maximum of B is shifted towards the crystal face through which beam *1* enters, i.e. the grating is 'attracted' to this face. This occurs in the absence of a dynamic grating in a PRC at the moment of entry of beam *1* and also when a DPCM is present and beam *1* in phase with beam *3* or beam *4* in phase with beam *2* enters the crystal. The grating B then shifts to the face through which the additional beam enters the crystal.

According to Ref. [5], the class 2 solutions can be obtained if a weak beam (*4*)is phase-shifted by π. However, there is a simpler way: beams *1*, *3*, and *2* are directed to a PRC without a dynamic grating; a steady state, described by a class 1 solution, is then established; the difference between the phases of beams *1* and *3* is altered by π; a state described by a class 2 solution is then obtained. The difference between the two classes of solutions consists, in particular, in the different direction of the shift of the maximum of B relative to its position in a DPCM. For a class 2 solution, the grating is 'repelled' by the face through which beam *1* enters (Fig. 4).

Class 1 solutions are obtained for any intensity of beam *1* since the procedure of 'crystal enlargement' can be continued to infinity. However, in the case of a class 2 solution, the intensity of beam *1* must be finite since the coupling constant $\gamma(l - l_{\mathrm{sh}})$ cannot be less than 2, because otherwise the condition for the formation of a DPCM is not obeyed. This is illustrated clearly in Fig. 6 where the dependences of l_{sh} on the intensity of beam *1* at the entry are plotted for several values of γl. The lower of the two branches of the solution shown in Fig. 6 is stable. It follows from Fig. 6 that the class 2 solutions are obtained in a limited range of $J_1(0)$. If $J_1(0)$ is

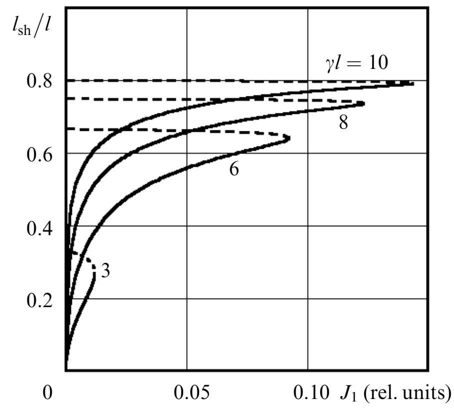

Figure 6. Dependences of the position (shift length) l_{sh} of the boundary of a double phase-conjugate mirror on the intensity of beam *1* when $\Phi = \pi$ subject to the condition that the sum of the intensities of beams *1* and *3* is equal to the intensity of beam *2*, i.e. $J_{\Sigma_1} = J_2(l)$.

outside this range, then a class 1 solution is obtained, but with $\Phi = \pi$.

5. Conclusions

A shifted dynamic grating in a PRC with a large coupling constant is localised in a limited part of this crystal and the degree of localisation increases with increase in the coupling constant. A change in the ratio of the intensities of beams *3* and *2* (for a DPCM) or in the ratio of the intensities and phase differences of beams *1* and *3* (for a conventional phase-conjugate mirror) can be used to control the spatial position of such a dynamic grating (Figs 3 and 4). This effect can be utilised for spatial positioning of beam *4* or of an additional beam *5*, diffracted by a grating formed in a PRC with a scheme of the kind shown in Fig. 7.

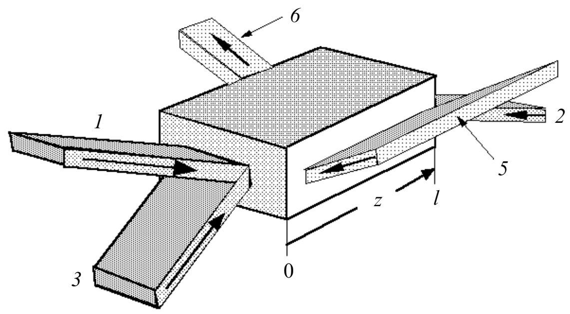

Figure 7. Optical scheme for spatial positioning of beam *5* by control of the spatial position of a dynamic grating formed as a result of four-wave mixing: (*1*–*3*) interacting beams (beam *4* is missing); *6* represents the output signal.

References

1. Cronin-Golomb M, Fischer B, White J O, Yariv A *IEEE J. Quantum Electron.* **QE-20** 12 (1984)
2. Odulov S G, Soskin M S, Khizhnyak A I *Lazery na Dinamicheskikh Reshetkakh* (Lasers with Dynamic Gratings) (Moscow: Nauka, 1990) p. 271
3. Odoulov S, Soskin M, Khizhnyak A *Optical Oscillators with Degenerate Four-Wave Mixing* (New York: Harwood Academic, 1991) p. 148

4. Cronin-Golomb M, in *Optical Phase Conjugation* (Eds M Gower, D Proch) (Berlin: Springer, 1993) p. 386
5. Krolikowski W, Shaw K D, Cronin-Golomb M *J. Opt. Soc. Am. B* **6** 1828 (1989)